

Vanishing Cosmological Constant by Gravitino-Dressed Compactification of 11D Supergravity

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ABSTRACT: We consider compactifications induced by the gravitino field of eleven dimensional supergravity. Such compactifications are not trivial in the sense that the gravitino profiles are not related to pure bosonic ones by means of a supersymmetry transformation. The basic property of such backgrounds is that they admit ψ -torsion although they have vanishing Riemann tensor. Thus, these backgrounds may be considered also as solutions of the teleparallel formulation of supergravity. We construct two classes of solutions, one with both antisymmetric three-form field, gravity and gravitino and one with only gravity and gravitino. In these classes of solutions, the internal space is a parallelized compact manifold, so that it does not inherit any cosmological constant to the external spacetime. The latter turns out to be flat Minkowski in the maximally symmetric case. The elimination of the cosmological constant in the spontaneously compactified supergravity seems to be a generic property based on the trading of the cosmological constant for parallelizing torsion.

KEYWORDS: 11-dimensional supergravity, compactification, gravitino, higher-dimensional theories

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1 Introduction

One of the most appealing ideas of modern theoretical physics is the possibility of existence of extra dimensions. Starting with the work of Kaluza and Klein, the idea that we live in a higher-dimensional environment has been incarnated in string and M-theory as well as in the brane-world scenarios. If this is the case and indeed there are extra dimensions, large or small, or if we live in a 4D hypersurface, remains to be seen. A necessary part of every higher-dimensional theory is how the theory compactifies, i.e., a four-dimensional world is attained. Technically, compactification proceeds through the classical solutions, that serve as a vacuum of the field equations and then by dimensional reduction the lower-dimensional effective theory is obtained [1].

In particular, the $N = 1$ eleven-dimensional supergravity [2] is a maximal theory that offers a rich foundation for studying compactifications and dimensional reduction to lower dimensions. There are many known compactifications of eleven-dimensional simple supergravity. For example, the Freund-Rubin [3] compactification $AdS_4 \times S^7$ or $AdS_7 \times S^4$ gives

rise to lower dimensional theories on AdS spaces, much studied due to AdS/CFT correspondence [4]. These AdS spaces are for instance holographic duals of gauge theories but cannot be considered as true four-dimensional vacua, mainly because they have a large negative cosmological constant. On the other hand, flat Minkowski four- or higher-dimensional vacua also exist in eleven-dimensional supergravity when the latter compactifies on Ricci-flat internal spaces. We may recall the 4D $\mathcal{N} = 8$ compactification on T^7 or the $\mathcal{N} = 1$ compactification on G_2 -holonomy manifolds, or even the $\mathcal{N} = 2$ compactification on $CY_3 \times S^1$. Thus, we see that there are supersymmetric compactifications with flat 4D Minkowski spacetime. One may ask of course if there are also non-supersymmetric compactifications with 4D Minkowski spacetime. For instance, compactification on a generic non-Ricci flat internal space, e.g $Q(p, q, r)$ [5], will give rise to an AdS_4 spacetime. The reason is that if the internal space is not Ricci-flat, it will have a positive cosmological constant (for a maximally symmetric 4D spacetime) which will give rise to a negative cosmological constant in 4D and therefore to AdS_4 . Definitely, it is possible to consider an internal space with a negative cosmological constant and finite volume (for example by quotienting the internal space by a discrete appropriate subgroup of its non-compact isometry group [6–9]). In this case, a de Sitter 4D vacuum emerges but with a huge cosmological constant.

Another possibility is to break supersymmetry by employing different boundary conditions for bosons and fermions. In a torus compactification for example, one may adopt antiperiodic boundary conditions for fermions and periodic for bosons along the circles of the torus. This will give a mass to the 4D gravitino of order the torus radii, which breaks supersymmetry and creates a cosmological constant proportional again to the size of the internal space. Thus, we see that although there are supersymmetric compactifications with flat 4D Minkowski vacuum, non-supersymmetric ones are not-known or rare in the best case. It should be stressed nevertheless, that in supersymmetric compactifications Minkowski spacetime is no longer a real vacuum as soon as supersymmetry is broken. Supersymmetry breaking produces a cosmological constant which shifts the vacuum from Minkowski to de Sitter. We may then state that compactifications of higher-dimensional supergravity, where only bosonic fields are turned on, cannot result in a 4D Minkowski vacuum as long as supersymmetry is broken. This problem is our motivation for searching for non-supersymmetric flat 4D vacua in 11D simple supergravity. We will argue that we may get over this problem by turning on fermionic fields, which in the case we are discussing, eleven-dimensional supergravity, amounts to allow for non-vanishing gravitino field.

The plan of the work is as follows: In section 2 we briefly review simple supergravity in eleven dimensions and in section 3 we describe the general procedure for extracting solutions. In section 4 we explicitly present the class of solutions with both bosonic and fermionic degrees of freedom and in 5 that with pure fermionic fields. In section 6 we examine the triviality of the obtained solutions, and finally in section 7 we conclude and we discuss our results.

2 Simple supergravity in eleven dimensions

Let us briefly recall $\mathcal{N} = 1$ simple supergravity in 11 dimensions [1, 2, 10]. The field content of the theory consists of the vierbein e^A_M , a Majorana anticommuting spin- $\frac{3}{2}$ field ψ_M and a completely antisymmetric 3-form gauge field A_{KLM} . The field equations for the eleven dimensional supergravity read [2]:

$$\Gamma^{RST} \hat{D}_S(\hat{\omega}) \psi_T = 0, \quad (2.1)$$

$$D_T(\hat{\omega}) \hat{F}^{TURS} = (24)^{-2} \epsilon^{MNPQVWXYUSR} \hat{F}_{MNPQ} \hat{F}_{VWXY}, \quad (2.2)$$

$$R_{TS}(\hat{\omega}) - \frac{1}{2} g_{TS} R(\hat{\omega}) = \frac{1}{24} \left[g_{TS} \hat{F}_{MNPQ} \hat{F}^{MNPQ} - 8 \hat{F}_{MNP} T^M_S \right], \quad (2.3)$$

where the super-covariant \hat{D}_S and covariant derivatives D_S are

$$\hat{D}_S(\hat{\omega}) \psi_T = D_S(\hat{\omega}) \psi_T + T_S^{MNPQ} \hat{F}_{MNPQ} \psi_T, \quad (2.4)$$

$$D_S(\hat{\omega}) \psi_T = \partial_S \psi_T + \frac{1}{4} \hat{\omega}_{SAB} \Gamma^{AB} \psi_T. \quad (2.5)$$

The notation

$$T^{SMNPQ} = (12)^{-2} \left(\Gamma^{SMNPQ} - 8 \Gamma^{[MNP} \eta^{Q]S} \right), \quad (2.6)$$

has been used whereas the super-covariant spin-connection and the super-covariant field strength are

$$\hat{\omega}_{MRS} = \omega_{MRS}(e^A_M) + \frac{1}{2} i (2 \bar{\psi}_M \Gamma_{[S} \psi_{R]} + \bar{\psi}_S \Gamma_M \psi_R), \quad (2.7)$$

$$\hat{F}_{MNPQ} = F_{MNPQ} - 3 \bar{\psi}_{[M} \Gamma_{NP} \psi_{Q]}, \quad (2.8)$$

respectively, with

$$F_{MNPQ} = 4 \partial_{[M} A_{NPQ]}. \quad (2.9)$$

It should be stressed that relation (2.7) leads to the non-vanishing effective torsion tensor given by

$$T^A_{MN} = -i \bar{\psi}_M \Gamma^A \psi_N. \quad (2.10)$$

In these expressions, early alphabet capital letters ($A, B, \dots = 0, \dots, 10$) are tangent space indices, while middle and late alphabet capital letters ($K, L, \dots = 0, \dots, 10$) are world indices. Moreover, the eleven dimensional tangent space metric η is mostly minus, i.e., $\eta_{AB} = \text{diag}(1, -1, \dots, -1)$. Finally, eleven-dimensional Γ -matrices are in the Majorana representation and they form a purely imaginary representation of the Clifford algebra in 11 dimensions. More details are given in the Appendix.

The supergravity equations (2.1)-(2.3) are invariant under the local supersymmetry transformations

$$\delta e^A_M = -i \bar{\epsilon} \Gamma^A \psi_M, \quad (2.11)$$

$$\delta \psi_M = D_M(\hat{\omega}) \epsilon - \frac{i}{144} (\Gamma^{NPQR} \epsilon + 8 \Gamma^{PQR} \delta^N_M) \hat{F}_{NPQR} \epsilon, \quad (2.12)$$

$$\delta A_{MNP} = \frac{3}{2} \bar{\epsilon} \Gamma_{[MN} \psi_{P]}, \quad (2.13)$$

where ϵ is the fermionic supersymmetry transformation parameter.

3 Compactification

The $N = 1$ supergravity in eleven dimensions [2] is a maximal theory and has offered a rich foundation for studying compactification mechanisms. The investigation of the latter has been limited to find solutions to the classical equations of motion where all fermionic fields (here gravitino) vanish. Especially, Freund-Rubin type of vacua [3] provide a natural framework and pave the way not only for the spontaneous compactification of eleven-dimensional supergravity but also for any higher-dimensional theory. However, this compactification mechanism where bosonic degree of freedom are excited, suffers from a huge cosmological constant, that cannot be eliminated or reduced [10]. Thus, we are led to explore the possibility of compactifications where in addition fermionic fields are allowed to be non-vanishing and lead to 4D flat Minkowski vacuum. For the case we are discussing, eleven-dimensional supergravity, this means that we will excite the gravitino field. However, the gravitino is an anticommuting Grassmann field and its classical limit vanishes, and therefore there cannot be a classical gravitino field. Nevertheless, it can form bilinears or quadrilinears and so on which have a classical interpretation. We recall that for an anticommuting charged fermion field $\Psi(x)$ satisfying the Dirac equation, there is a classical electron density field $W(x, p)$ given by the Wigner transform of the bilinear $\bar{\Psi}(x)\Psi(y)$, which is measurable and plays a role in semiconductor modeling. In a sense, this is equivalent of solving the equations of motion and taking expectation values of the fields involved. Thus, interpreting the gravitino as a fermionic quantum field, its vacuum expectation value will vanish,

$$\langle \psi^A \rangle = 0, \quad (3.1)$$

which however does not imply that the vacuum expectation values of various bilinears should vanish as well. For example (3.1) may hold but certain of its possible bilinears may be non-zero, [11, 12], i.e.,

$$\langle \psi^A \Gamma^{B_1} \dots \Gamma^{B_n} \psi^C \rangle \neq 0. \quad (3.2)$$

The non-vanishing of the vacuum expectation values of some of the gravitini bilinears has been considered before [13], [14] in order to find flat Minkowski vacua. Here we construct explicitly solutions that realize (3.1), (3.2) and we describe compactifications of the internal space in such a way that the four-dimensional cosmological constant vanishes.

We are interested in finding solutions of the supergravity equations (2.1)-(2.3), corresponding to a direct product of a seven-dimensional compact space with a four-dimensional Minkowski spacetime. The standard ansatz for the compactification metric should then be

$$\begin{aligned} \langle g_{\mu\nu} \rangle &= \eta_{\mu\nu}, \\ \langle g_{mn} \rangle &= g_{mn}, \\ \langle g_{m\nu} \rangle &= \langle g_{\mu n} \rangle = 0. \end{aligned} \quad (3.3)$$

Eleven-dimensional indices have been split as

$$\begin{aligned} M &= (\mu, m), \\ \mu &= 0, 1, 2, 3, \text{ (4D world indices)} \\ m &= 1, 2, \dots, 7 \text{ (7D world indices)}. \end{aligned}$$

For the super-covariant 4-form field strength \hat{F}_{KLMN} , we would like to have

$$\langle \hat{F}_{KLMN} \rangle = 0. \quad (3.4)$$

This choice may seem rather ad hoc but it is dictated by the fact that \hat{F}_{KLMN} transforms covariantly under supersymmetry and it is the generalization of the $F_{KLMN} = 0$ in the pure bosonic case. Definitely the ansatz (3.4) does not imply that fermion bilinears and bosonic four-form field strength vanish independently. As we will see there are cases where both fermionic bilinears and bosonic field strength is not vanishing and they just cancel each other. For the gravitino field we consider the splitting

$$\psi^A = (\psi^\alpha, \psi^a). \quad (3.5)$$

From the four dimensional point of view ψ^A gives rise to 8 spin 3/2 gravitini ψ^α and $8 \times 7 = 56$ spin 1/2 fermions ψ^a . The next step is to ensure that the torsion is non vanishing and fully antisymmetric only in the internal space, while it is zero in the external 4D spacetime. In the opposite case, a non-zero torsion in 4D, would require a non-flat metric as torsion would act as source in the right-hand side of the 4D Einstein equations (2.3). In fact the latter can be written as

$$R_{MN}(\hat{\omega}) = 0, \quad (3.6)$$

which is clearly solved by

$$\hat{\omega}_{ABC} = 0. \quad (3.7)$$

This means that the spin connection is given entirely in terms of the gravitino bilinear

$$\omega_{ABC}(e^A_M) = -\frac{1}{2}i (2\bar{\psi}_A \Gamma_{[C} \psi_{B]} + \bar{\psi}_C \Gamma_A \psi_B). \quad (3.8)$$

As a result, the spin connection may be written simply in terms of the torsion (2.10) as

$$\omega_{ABC} = -(T_{ABC} - T_{ACB} + T_{CAB}) \quad (3.9)$$

In other words, the connection is just the Weitzenböck connection

$$\Gamma^M_{KL} = E^K_A \partial_K E^A_L. \quad (3.10)$$

This corresponds exactly to teleparallelism of simple supergravity: since super-connection is zero, Riemann curvature tensor vanishes and one may define parallel transport over finite distances and not only in infinitesimal neighborhood. However, parallelograms do

not close under parallel transport, a manifestation of torsion. This is a known fact for supersymmetry [15] (and general relativity as well) and has been considered so far as a curiosity rather than a fact of fundamental importance. Here we use it as a tool generating technique for finding solutions [16].

The vanishing of the supercovariant connection (3.7) simplifies significantly the equations of motion. In particular, the gravitino equation (2.1) turns now to be

$$\Gamma^{ABC}\partial_B\psi_C = 0. \quad (3.11)$$

Clearly solutions to the above equation are provided by constant anticommuting Majorana fermions ψ_A , i.e.,

$$\psi_A(x) = \psi_A. \quad (3.12)$$

The equation for the supercovariant field strength (2.2) is also satisfied due to (3.4). Finally the gravitational equation (2.3) reduces to (3.6), which is split into

$$R_{\mu\nu}(\hat{\omega}) = 0, \quad (3.13)$$

$$R_{mn}(\hat{\omega}) = 0, \quad (3.14)$$

and which are always satisfied since $\hat{\omega} = 0$. However, since we want 4D Minkowski vacuum, (3.13) should be solved by

$$g_{\mu\nu} = \eta_{\alpha\beta}e_\mu^\alpha e_\nu^\beta = \text{diag}(1, -1, 1, -1), \quad \omega_{\mu\alpha\beta}(e_\mu^\alpha) = 0, \quad T^\alpha{}_{\mu\nu} = 0, \quad (3.15)$$

where e_μ^α is the four-dimensional vierbein. In other words, we are looking for flat Minkowski metric with vanishing connection and torsion.

What remains to be solved is actually (3.14). In the following we will separately explore two classes of solutions in the context of the above formulation. In particular, the first class contains both bosonic (F_{KLMN}) and fermionic (ψ_M) degrees of freedom, while the second class contains only fermionic (ψ_M) fields.

A final comment concerns supersymmetry. The supersymmetry transformations in the present framework take the form

$$\delta e^A{}_M = -i\bar{\epsilon}\Gamma^A\psi_M, \quad (3.16)$$

$$\delta\psi_A = \partial_A\epsilon, \quad (3.17)$$

$$\delta A_{ABC} = \frac{3}{2}\bar{\epsilon}\Gamma_{[AB}\psi_{C]}. \quad (3.18)$$

Supersymmetry is preserved whenever supersymmetry fermionic charges annihilate the vacuum. This is equivalent to consider the supersymmetry parameter ϵ as a commuting spinor. Then, contrary to the bosonic case where all bosonic variations vanish (due to vanishing of fermionic field), here all fermionic shifts vanish and one should consider the bosonic ones. The latter do not vanish in general due to non-zero bilinears.

4 Freund-Rubin compactification with zero cosmological constant

In this section we will consider a Freund-Rubin ansatz for the three-form field in the external space [3]. In particular, the four-form field strength takes the form

$$F_{\alpha\beta\gamma\delta} = 6m_0 \epsilon_{\alpha\beta\gamma\delta}, \quad \alpha, \beta, \dots = 0, 1, 2, 3, \quad (4.1)$$

$$F_{KLMN} = 0 \quad \text{otherwise}, \quad (4.2)$$

where m_0 is a constant. With a vanishing gravitino field, the field equations for the three-form field and gravity are reduced to the solution of

$$R_{\mu\nu} = -12m_0^2 g_{\mu\nu} \quad (4.3)$$

$$R_{mn} = 6m_0^2 g_{mn}. \quad (4.4)$$

For a maximally symmetric background we find the celebrated $AdS_4 \times S^7$ vacuum.

We will now turn on a gravitino field and look for solutions of the form $M^4 \times B^7$ where M^4 is a 4D Minkowski spacetime and B^7 is a 7D compact manifold. Below we will present three different solutions for the gravitino field, that solves the supergravity fields equations and provide a vacuum of this form.

4.1 Solution $M^4 \times S^7$

To proceed, let us consider the following form of the gravitino ψ_M

$$\psi^\alpha = (H^\alpha \otimes \theta), \quad (4.5)$$

$$\psi^a = (J \otimes \theta^a), \quad (4.6)$$

with

$$\begin{aligned} J &= (a, a, c, c)^T, \\ H^\alpha &= \gamma^5 \gamma^\alpha J, \\ \theta &= (1, 0, 0, 0, 0, 0, 0)^T, \\ \theta^a &= \tau^a \theta. \end{aligned}$$

The parameters a, c are constant real Grassmann variables, which guarantee that the gravitino satisfies the Majorana condition [17],

$$\psi_M^\dagger \Gamma^0 = \psi_M^T C. \quad (4.7)$$

Moreover, it can easily be verified that

$$\bar{\psi}_{[\alpha} \Gamma_{\beta\gamma} \psi_{\delta]} = (4iac) \epsilon_{\alpha\beta\gamma\delta}, \quad (4.8)$$

so that the condition $\hat{F}_{KLMN} = 0$ gives

$$F_{\alpha\beta\gamma\delta} = (12iac) \epsilon_{\alpha\beta\gamma\delta}. \quad (4.9)$$

Thus, by comparing it with (4.1), we find that

$$m_0 = 6i ac. \quad (4.10)$$

Then also all conditions at the beginning of section (3) hold. In particular, by using the relations

$$\begin{aligned} \theta^T \tau_a \theta &= 0, \\ \theta^T \tau_a \tau_b \theta &= -\delta_{ab}, \\ \theta^T \tau_a \tau_b \tau_c \theta &= -a_{abc}, \end{aligned}$$

where a_{abc} are the octonionic stracture constants

$$a_{abc} = -\theta^T \tau_{abc} \theta, \quad (4.11)$$

we find that the torsion is

$$T_{abc} = \frac{i}{2} \bar{\psi}_a \Gamma_b \psi_c = -\frac{m_0}{3} a_{abc}, \quad a, b, c = 1, \dots, 7 \quad (4.12)$$

$$T_{ABC} = 0, \quad \text{otherwise} \quad (4.13)$$

Therefore, the torsion is fully antisymmetric in the internal space and it vanishes anywhere else. It leads also (4.12) to the spin connection

$$\omega_{abc}(e^a_m) = -\frac{m_0}{3} a_{abc}, \quad (4.14)$$

$$\omega_{ABC} = 0, \quad \text{otherwise.} \quad (4.15)$$

In particular, the spin connection vanishes for the external spacetime

$$\omega_{\alpha\beta\gamma}(e^\alpha_\mu) = 0. \quad (4.16)$$

Thus, since the spin connection (4.16) vanishes, the external space can be chosen to be a four dimensional Minkowski spacetime (M^4)

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad (4.17)$$

$$R_{\mu\nu}(e^\alpha_\mu) = 0. \quad (4.18)$$

Concerning the internal space, we recall the well known result that when the spin connection has the form (4.14) a solution to eq.(3.14) is provided by the parallelized seven-sphere (S^7) and thus [13, 18]

$$R_{mn}(e^a_m) = 0. \quad (4.19)$$

In summary, the 11-dimensional spacetime splits as $M^4 \times S^7$.

4.2 Solution $M^4 \times S^3 \times T^4$

We may continue looking for other compactifying spaces with an ansatz similar to (4.5), (4.6), where this time θ^a is of the form

$$\theta^a = (\tau^1\theta, \tau^2\theta, \tau^3\theta, 0, 0, 0, 0). \quad (4.20)$$

Then the torsion turns out to be

$$T_{\bar{a}\bar{b}\bar{c}} = (-2iac)\epsilon_{\bar{a}\bar{b}\bar{c}}, \quad \bar{a}, \bar{b}, \bar{c} = 1, 2, 3 \quad (4.21)$$

$$T_{ABC} = 0, \quad \text{otherwise.} \quad (4.22)$$

This is just the parallelizing torsion for the three-sphere (S^3). We also have

$$F_{\alpha\beta\gamma\delta} = (12iac)\epsilon_{\alpha\beta\gamma\delta}, \quad (4.23)$$

in order to have $\hat{F}_{KLMN} = 0$. Einstein equations are then simply

$$R_{\mu\nu} = 0, \quad (4.24)$$

$$R_{\bar{m}\bar{n}} = 0, \quad (4.25)$$

$$R_{mn} = 0, \quad (4.26)$$

where $m, n = 1, \dots, 4$ and $\bar{m}, \bar{n} = 1, 2, 3$. The solution to (4.24) is just 4D Minkowski space M^4 , the solution to (4.25) is provided by the parallelized S^3 and the maximally symmetric solution to (4.26) is a flat torus T^4 .

4.3 Solution $M^4 \times T^7$

There is another class of solutions where the non-vanishing components of the gravitino is along four dimensions only, i.e.,

$$\psi^\alpha = (H^\alpha \otimes \theta), \quad (4.27)$$

$$\psi^a = 0. \quad (4.28)$$

A gravitino field of this sort leads to

$$\omega_{ABC}(e^A_M) = 0, \quad (4.29)$$

and the torsion is always zero. This kind of solution implies a Minkowski external spacetime (M^4) and a flat 7D torus T^7 -geometry in the maximally symmetric case. In summary the 11-dimensional spacetime is of the form $M^4 \times T^7$.

4.4 Supersymmetry

Let us now investigate the supersymmetry transformations of the above three solutions. The first two solutions, namely of subsections 4.1 and 4.2, have completely broken supersymmetry, that is there is not any spinorial parameter that makes all field-shifts to vanish.

However, for the solution in subsection 4.3 there is one such spinorial parameter ϵ that makes all field-shifts to vanish, namely

$$\epsilon = T \otimes \theta, \quad (4.30)$$

where

$$T = (-a, a, c, -c)^T. \quad (4.31)$$

It is then straightforward to verify that

$$\delta e^A_M = 0, \quad (4.32)$$

$$\delta \psi_M = 0, \quad (4.33)$$

$$\delta A_{MNP} = 0 \quad (\text{up to a gauge transformation}). \quad (4.34)$$

for supersymmetry parameters ϵ given in (4.30). However, as there are non-vanishing fermionic fields, Lorentz symmetry is broken.

5 Pure fermionic solutions with zero cosmological constant

We will consider here a class of solutions where no antisymmetric three-form field is turned on. As a result, the only non vanishing fields are the graviton and the gravitino. We will present two different solutions for the gravitino field, that will lead to Minkowski vacuum and thus to a vanishing cosmological constant.

5.1 $M^4 \times S^7$

Assume a gravitino of the form

$$\psi^\alpha = (0, 0, 0, H^3 \otimes \theta), \quad (5.1)$$

$$\psi^a = (J \otimes \theta^a), \quad (5.2)$$

where

$$\begin{aligned} J &= (a, a, c, c)^T, \\ H^3 &= \gamma^5 \gamma^3 J, \\ \theta &= (1, 0, 0, 0, 0, 0, 0)^T, \\ \theta^a &= \tau^a \theta. \end{aligned} \quad (5.3)$$

The constants a and c are again anticommuting Grassmann variables and the Majorana condition is still valid. However, contrary to the previous class of solutions in (4.9), the condition $\hat{F}_{KLMN} = 0$ leads to

$$F_{ABCD} = 0, \quad (5.4)$$

since now

$$\bar{\psi}_{[M} \Gamma_{NP} \psi_{Q]} = 0. \quad (5.5)$$

Note that the vanishing of the field strength is realized in the whole spacetime, including the external one, that is $F_{\alpha\beta\gamma\delta} = 0$. Moreover, for the torsion we find

$$T_{abc} \equiv \frac{i}{2} \bar{\psi}_a \Gamma_b \psi_c, \quad a, b, c = 1, \dots, 7 \quad (5.6)$$

$$T_{ABC} = 0, \quad \text{otherwise}, \quad (5.7)$$

so that the spin connection turns out to be

$$\omega_{abc}(e_m^a) = -(2iac)a_{abc}, \quad (5.8)$$

$$\omega_{ABC} = 0, \quad \text{otherwise}. \quad (5.9)$$

It is obvious then that the internal space is compactified to a parallelized seven sphere, and the external space turns out to be a four dimensional Minkowski space-time. That is the 11D spacetime splits as $M^4 \times S^7$.

5.2 $M^8 \times S^3$

The second solution which we can get with only fermionic condensates contributing to the ground state, is given by

$$\psi^\alpha = (0, 0, 0, H^3 \otimes \theta), \quad (5.10)$$

$$\psi^a = (J \otimes \tau^1 \theta, J \otimes \tau^2 \theta, J \otimes \tau^3 \theta, 0, 0, 0, 0). \quad (5.11)$$

The conventions are also given by (5.3), but now we have a parallelized three sphere S^3 and an 8D Minkowski spacetime M^8 . That is the 11D spacetime compactifies to $M^8 \times S^3$. Of course M^8 can further be compactified on a 4D torus T^4 to $M^4 \times T^4$. This corresponds to a $M^4 \times T^4 \times S^3$ compactification of the 11D theory.

5.3 Supersymmetry

Let us now consider the supersymmetry transformations of the solutions of subsections 5.1 and 5.2. It is possible to find the existence of three supersymmetries if we use the following explicit form for the supersymmetry parameter:

$$\epsilon_{(i)} = T_{(i)} \otimes \theta, \quad (5.12)$$

where

$$\begin{aligned} T_{(1)} &= (c, c, 0, 0)^T, \\ T_{(2)} &= (0, 0, a, a)^T, \\ T_{(3)} &= (a, a, -c, -c)^T. \end{aligned} \quad (5.13)$$

Then for both solution subclasses:

$$\delta e^A_M = 0, \quad (5.14)$$

$$\delta \psi_M = 0, \quad (5.15)$$

$$\delta A_{MNP} = 0 \text{ (up to a gauge transformation)}. \quad (5.16)$$

6 Triviality of the Fermionic Vacua

One question concerns the triviality of the vacuum solutions found above. Triviality here has the meaning of relating these vacua with non-zero fermionic fields to pure bosonic backgrounds with vanishing fermionic fields by means of a supersymmetry transformation. For example, for the gravitino profile (4.5),(4.6), triviality means that there is Majorana spinor ϵ such that

$$\psi_\alpha = D_\alpha(\hat{\omega})\epsilon, \quad \psi_a = D_a(\hat{\omega})\epsilon. \quad (6.1)$$

If (6.1) is valid, then, ψ_α, ψ_a can be shifted to zero ($\psi_\alpha = 0, \psi_a = 0$) by means of a supersymmetry transformation, resulting in a pure bosonic background by a corresponding shift of the veilbein and the three-form field.

Let us suppose that the solution in section 4.1 is trivial and indeed ψ_α, ψ_a can be written as in (6.1). This means that there is a non-zero ϵ which we write as

$$\epsilon = J \otimes \lambda \quad (6.2)$$

and thus, λ should satisfy for example

$$D_a(\hat{\omega})\lambda = \tau_a\theta. \quad (6.3)$$

Since $\hat{\omega} = 0$, we find that λ should satisfy the condition

$$\tau^m \partial_m \lambda = 7\theta. \quad (6.4)$$

Acting with $\tau^m \partial_m$ on both sides of the above equation, we get

$$\square \lambda_i = 0, \quad (6.5)$$

where λ_i are c-functions in the expansion of λ in terms of a Grassmann base (a_i) , $\lambda = \sum_i \lambda_i a_i$. Multiplying (6.5) with λ^* and integrating over the whole S^7 , we find that $\lambda_i = \text{const.}$ and similarly ϵ is a constant spinor. Then clearly there is no solution with constant ϵ to (6.3) and thus the solution for (ψ_α, ψ_a) is not trivial.

Proceeding similarly, one may prove that none of the solutions presented is trivial except the $M^4 \times T^7$ of section (4.3). The latter has no paralelizing torsion at all and it can be seen that the gravitino is pure gauge (connected to $\psi_M = 0$ by a susy transformation).

Below we review the solutions found with the help of the following table

fermionic v.e.v.s	bosonic v.e.v.s	geometry	triviality
Majorana	yes	$M^4 \times S^7$	no
Majorana	yes	$M^4 \times S^3 \times T^4$	no
Majorana	yes	$M^4 \times T^7$	yes
Majorana	no	$M^4 \times S^7$	no
Majorana	no	$M^8 \times S^3$	no

7 Conclusions

As has been stressed above, all known compactifications of higher dimensional theories (supersymmetric or not) inherit a cosmological constant to 4D spacetime. Although this is not the case for a supersymmetric compactification, like a *CY* or any other appropriate internal space, a cosmological constant emerges as soon as supersymmetry is broken. In the latter case, the value of the cosmological constant is determined by the gravitino mass, which is nevertheless non-zero (or many orders of magnitude away of the reported value of the cosmological constant). On the other hand, for non-supersymmetric compactifications the cosmological constant value is determined by the size of the internal space. In any case, standard compactifications produce a huge non-zero cosmological constant. The word “standard”, refers here to the well established procedure of putting all fermionic fields to zero and allowing for non-zero bosonic fields to determine the vacuum of the theory. There is nothing wrong with this as long as a mechanism to reduce the cosmological constant is found. But one would like to explore the possibility of other compactifications which do not inherit a cosmological constant in first place.

In the search for such compactifications, we are led to the conclusion that one may allow for non-vanishing fermionic fields, which in the case of 11D supergravity translates into a non-vanishing gravitino field. Of course, in this case, one faces the fact that there is no classical limit of a fermionic field, strictly speaking, as the latter is an anticommuting object. However, recalling that although a fermion field cannot be classical, fermion bilinears, quadrilinears and so on may have classical limits and therefore may be observed. Hence, the way we are to interpret our findings is that the vacuum of the theory is determined by the expectation values of the fields involved and their local products. Thus, although we find a non-vanishing profile for the 11D gravitini, the classical vacuum is determined by its expectation value of itself and its local products like $\langle \bar{\psi}_M \psi^M \rangle$, $\langle \bar{\psi}_M \Gamma^M \psi_N \rangle$ etc. In this way, the backgrounds found do not produce a cosmological constant in 4D, and thus a Minkowski vacuum is possible, although supersymmetry is clearly broken. As usual nothing comes for free. The cost paid in our case is the appearance of ψ -torsion instead of a cosmological constant (or curvature). However, the torsion appears in the internal space and it is responsible for the parallelism of the internal space and the flatness of the external, allowing this way Minkowski spacetime as 4D vacuum. Indeed in all backgrounds found here, it is in fact the explicit solution to the Majorana condition on the gravitino that provides the parallelizing torsion in the internal space for each case, apart from the case $M^4 \times T^7$, where no parallelization exists. The latter is trivial in any case as it is connected by a supersymmetry transformation to a pure bosonic background. In fact appropriate supersymmetry transformation turns $\psi_M = 0$ while keeping the $M^4 \times T^7$ geometry without vanishing three-form field.

Finally, we expect that more solutions may exist in the spirit of section 3 for eleven dimensional supergravity as well as for lower-dimensional ones, which should be studied.

Acknowledgments

This work is supported by the NTUA PEVE-2009 programme for basic research.

Appendix

A Clifford Algebra

The usual, pure imaginary, Majorana representation of the Clifford algebra in 11D Kaluza-Klein supergravity is [2, 17]:

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}\mathbf{1}_{32}, \quad (\text{A.1})$$

where $\mathbf{1}_n$ is the $n \times n$ unit matrix. $\Gamma^{M_1 M_2 \dots M_N}$ as usual represents the full antisymmetrized product of N Γ matrices. One convenient representation of the Γ -matrix algebra is the following:

$$\begin{aligned} \Gamma^A &= (\Gamma^\alpha, \Gamma^a), \\ \Gamma^\alpha &= \gamma^\alpha \otimes \mathbf{1}_8, \\ \Gamma^a &= \gamma^5 \otimes \tau^a, \\ C_{11} &= C_4 \otimes C_7 = \gamma^0 \otimes \mathbf{1}_8 = \Gamma^0, \end{aligned}$$

where

$$\begin{aligned} A &= (\alpha, a), \\ \alpha &= 0, 1, 2, 3, \text{ (tangent spacetime)} \\ a &= 1, 2, \dots, 7 \text{ (tangent internal space)} \end{aligned}$$

and

$$\begin{aligned} (\tau^a)_{bc} &= a_{abc}, \\ (\tau^a)_{0b} &= -(\tau^a)_{b0} = \delta_{ab}, \\ (\tau^a)_{00} &= 0. \end{aligned}$$

In the above expressions, the τ matrices form a real representation of the seven dimensional Clifford algebra:

$$\{\tau^a, \tau^b\} = -2\delta^{ab}\mathbf{1}_8. \quad (\text{A.2})$$

Additionally, a_{abc} are the octonionic algebra full antisymmetric structure constants [17], which vanish apart from the following entries:

$$a_{147} = a_{123} = -a_{156} = a_{257} = a_{246} = a_{367} = -a_{345} = 1. \quad (\text{A.3})$$

The four dimensional γ matrices are considered in the Majorana representation and form a pure imaginary representation of the clifford algebra in a minkowski space:

$$\{\gamma^\alpha, \gamma^\beta\} = 2\eta^{\alpha\beta}\mathbf{1}_4, \quad (\text{A.4})$$

where $\eta^{\alpha\beta}$ has the form $\text{diag}[1,-1,-1,-1]$. They explicitly have the form:

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad (\text{A.5})$$

$$\gamma^1 = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}, \quad (\text{A.6})$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \quad (\text{A.7})$$

$$\gamma^3 = \begin{pmatrix} 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix}, \quad (\text{A.8})$$

$$\gamma^5 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}. \quad (\text{A.9})$$

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